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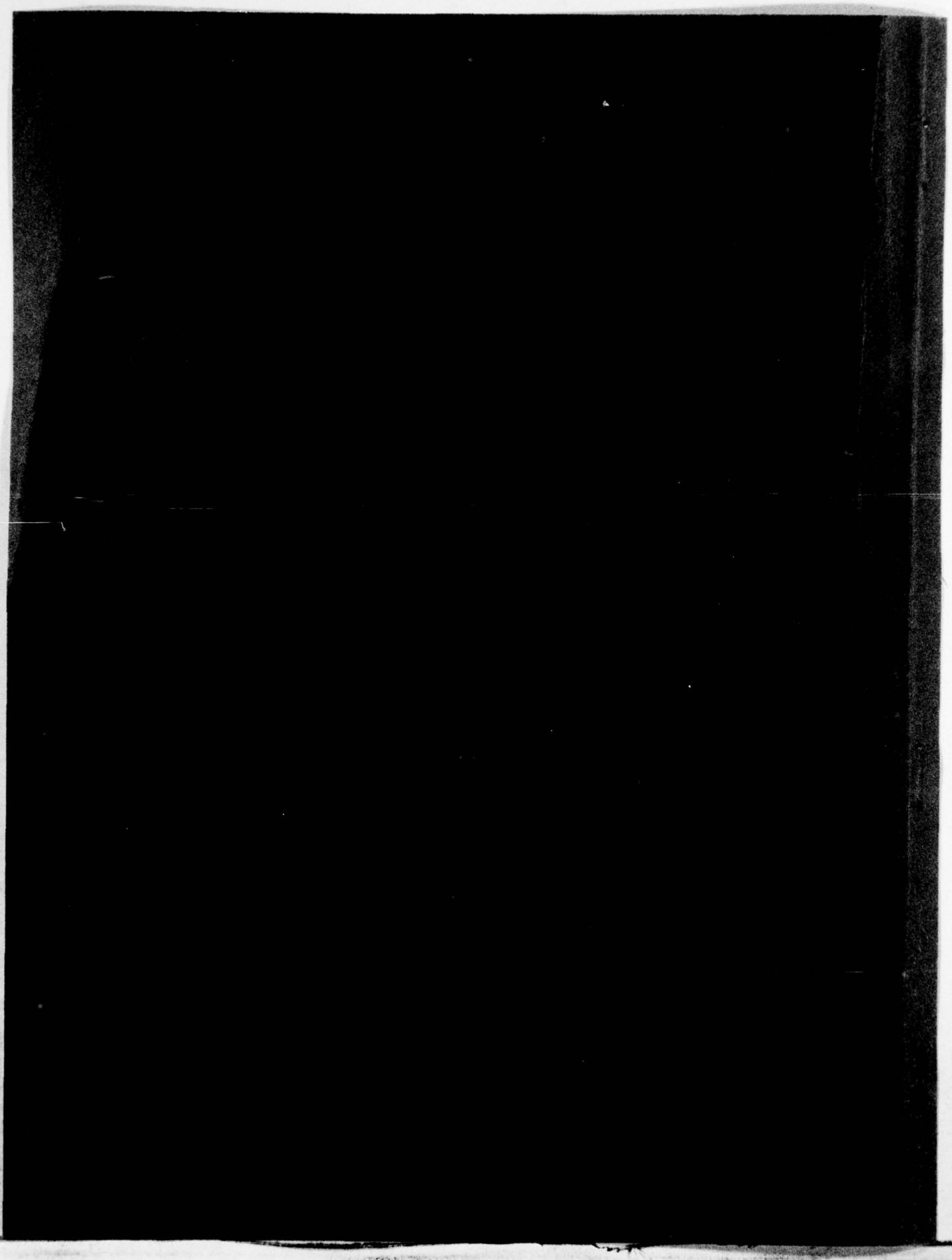

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This report documents a continuing investigation of two recent programs for the solution of AX = B where A is symmetric and sparse: the Yale Sparse Matrix Package and the Munksgaard subroutines. These two programs compute direct solutions in core using triangular decomposition and Gaussian elimination, respectively. Their performance is compared with that of an out-of-core Cholesky algorithm equation solver, CSKYDG2.

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IMAGE is designed to run quickly and efficiently. Speed is attained by minimizing the number of requests required to perform a function. Efficiency results from programming each transaction as a separate module, called a task. These tasks are loaded into core memory only as needed. Virtual memory techniques also save core storage.

The program is available for general use on the Tektronix 4014 and the CDC IGS 274 terminals linked to the DTNSRDC CDC 6700 and 6400 computers.

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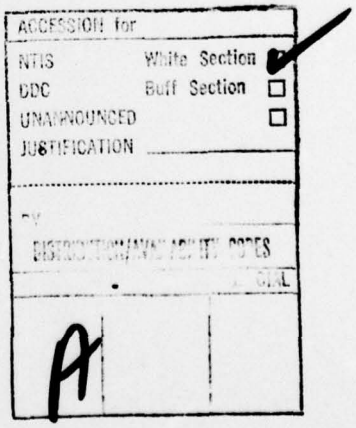

ABSTRACT

This report documents a continuing ivestigation of two recent programs for the solution of AX = B where A is symmetric and sparse: the Yale Sparse Matrix Package and the Munksgaard subroutines. These two programs compute direct solutions in core using triangular decomposition and Gaussian elimination, respectively. Their performance is compared with that of an out-of-core Cholesky algorithm equation solver, CSKYDG2.

As would be expected, the two in-core equation solvers are much faster than CSKYDG2. The Yale symmetric subroutines range up to six times faster than the Munksgaard subroutines. All three equation solvers provide the same degree of accuracy. However, the two in-core equation solvers require such enormous amounts of core storage that their use is not recommended on the CDC 6000 series of computers in their present form. While CSKYDG2 requires less core storage, its runs on the CDC 6400 cost more because of the repeated use of the random access storage capability.

INTRODUCTION

One of the long range projects of the Computation, Mathematics, and Logistics Department has been the development of mathematical subroutines suitable for use in the computer-aided structural analysis of ships. Many unrelated efforts in both government and industry have resulted in computer programs that treat particular classes of structural problems. These programs often involve the solution of similar mathematical problems but, since the solutions are reached independently, the efficiency and accuracy of the various algorithms used may vary greatly. The need to coordinate these diverse efforts, to develop improved methods of more general applicability, and to produce more comprehensive programs for solving Navy structural problems became obvious. A project was therefore established to coordinate research efforts involving mathematical and computational methods in the area of structural mechanics and to integrate the work of mathematicians, computer specialists, and structural engineers in this field.

The present considerable interest in the finite element approach to structural analysis is evidenced by the widespread use of NASTRAN (NAsa

STRuctural ANalysis program) and other such programs. According to the NASTRAN Theoretical Manual,[1*] "From a theoretical viewpoint, the formulation of a static structural problem for solution by the displacement method is completely described by the matrix equation KU=P." Thus, there is a need for accurate efficient computer subroutines capable of solving these large sparse positive definite systems of simultaneous linear equations. However, the order of K is often so large that, even when advantage is taken of K's symmetry and banded structure, it is not feasible and sometimes not even possible, to store K in the core memory of a computer.

This report continues the investigation[2] of two recently developed programs for solving KU=P, where K is a matrix of very large order. It documents the testing of these programs together with a third program[3] in order to compare their performances in solving large order systems taken from the finite element approach to structural analysis.[4]

THE PROGRAMS

The following programs are considered in this report:

- The Munksgaard collection of subroutines[5] implements Duff's algorithm for solving sparse symmetric systems. It can obtain a stable decomposition of K in both the definite and indefinite cases.[6] This program was developed at the Technical University of Denmark and the Atomic Energy Research Establishment of the United Kingdom.
- The Yale Sparse Matrix Package[7,8] is a collection of subroutines which can be used to solve both symmetric and non-symmetric systems. It uses algorithms developed by Eisenstat, et al.[7,8] Limitations are placed on the distribution and use of these programs by the Department of Computer Science, Yale University.
- CSKYDG2 is an out-of-core Cholesky algorithm equation solver developed by the present author.[3] It makes use of auxiliary storage via the random access capabilities of the CDC 6000 series of computers.

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*A complete listing of references is given on page 9.

THE TEST EXAMPLES

The table consists of the tabulated solution times for AX = B where A is a matrix constructed from the connection tables of Everstine.[4*] We construct A in the following manner. Let c be some positive constant. The off-diagonal non-zero elements of A are then set equal to -c. If there are ℓ such off-diagonal elements in a given row of A, then the diagonal element of that row is set equal to $(\ell+1)c$. This ensures that A is positive definite. Next set all the coordinates of B, the right-hand side of the system, equal to c. Then all coordinates of the solution vector X are equal to 1.

TABLE NOTATION

The computation described in this report was done in the spring of 1978 on the DTNSRDC CDC 6400 with 844 disk drives using the FTN 4.6+433 FORTRAN compiler and the NOS1BE 1.0 F+34 operating system. The times, given in terms of CDC 6400 CPU seconds, were obtained using the timing subroutine SECOND.

The column headings are defined as follows:

N - the order of the system

M - the system bandwidth

MATRIX DENSITY (%) - the ratio of the number of non-zero elements to N^2 multiplied by 100

INDANL TIME - the time required for the INDANL subroutine of the Munksgaard program to permute and factor the coefficient matrix A thus

$$B = PAP^T$$

$$B = (E+L)D(E+L^T)$$

where P is a permutation matrix chosen so as to "satisfy a stability criterion and have the least possible 'sparsity-cost'",[5] D and E are symmetric block diagonal matrices whose blocks are of order 1 or 2, and L is strictly lower triangular.

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*NOTE: The connection tables of Everstine are available locally on a CDC 6400 permanent file.

INDOPR TIME - the time required for the INDOPR subroutine of the Munksgaard program to solve the system

$$(E+L)D(E+L^T)P\,Y = P\,b$$

by forward and backward substitutions and obtain X from

$$X = P^T Y$$

TOTAL TIME - the sum of INDANL TIME and INDOPR TIME.

ORDV TIME - the time required for the ORDV subroutine from the Yale package to reorder the system using the maximum degree ordering algorithm with threshold search.[7]

SDRV TIME - the time required for the SDRV subroutine from the Yale package to compute the LDL^T decomposition of the permuted matrix A and obtain the solution X of the original system AX = B using forward and backward substitution as done previously.

TOTAL TIME - the sum of ORDV TIME and SDRV TIME.

SETUP TIME - the time required by the SETUP preprocessor subroutine for CSKYDG2 to read the coefficient matrix A from tape and pack it in blocks in random access storage.

CSKYDG2 TIME - the time required by the CSKYDG2 subroutine to factor the coefficient matrix A using the Cholesky decomposition and obtain the solution of

$$LL^T X = B$$

by forward and backward substitution.

TOTAL TIME - the sum of SETUP TIME and CSKYDG2 TIME.

TABLE 1 - SOLUTION TIMES

N,M	Matrix Density (percent)	DENMARK Program			YALE Program			CSKYDG2 Program		
		INDANL Time	INDOPR Time	Total Time	ORDV Time	SDRV Time	Total Time	Setup Time	CSKYDG2 Time	Total Time
59,26	7.67	0.253	0.017	0.270	0.149	0.067	0.216	0.453	0.530	0.983
66,45	7.35	0.187	0.016	0.204	0.135	0.052	0.187	0.553	0.680	1.233
72,13	4.28	0.161	0.018	0.179	0.115	0.052	0.167	0.634	0.326	0.960
87,64	7.15	0.462	0.027	0.489	0.240	0.109	0.349	0.919	2.759	3.676
162,157	4.50	1.811	0.065	1.876	0.639	0.363	1.002	2.848	3.420	6.268
193,63	9.38	19.870	0.196	20.066	2.743	1.823	4.566	4.055	8.321	12.376
198,37	3.55	1.557	0.071	1.628	0.609	0.328	0.937	4.294	4.665	8.959
209,185	3.99	3.824	0.106	3.930	1.163	0.604	1.767	4.789	23.140	27.929
221,188	3.34	2.908	0.097	3.005	0.855	0.481	1.336	6.413	20.138	26.551
234,49	1.52	2.217	0.057	2.274	0.310	0.183	0.493	5.886	2.458	8.344
245,116	2.43	2.137	0.087	2.224	0.693	0.373	1.066	4.286	4.605	8.893
307,64	2.68	18.393	0.230	18.623	2.179	1.763	3.942			
310,29	2.55	6.006	0.162	6.168	1.472	0.947	2.419	10.285	2.359	12.644
346,329	2.69	18.119	0.233	18.352	2.962	2.024	4.986	12.933	12.604	25.537
361,13	2.27	17.076	0.253	17.329	2.145	1.588	3.733			
491,357	2.03	15.433	0.259	15.692	2.479	1.683	4.162			
492,436	1.30	3.685	0.161	3.846	1.268	0.721	1.989			
503,453	2.38	46.481	0.462	46.943	5.805	5.111	10.916			
512,74	1.34	7.894	0.198	8.092	1.985	1.119	3.104	32.686	8.639	41.325
592,260	1.46	21.883	0.361	22.244	3.644	2.450	6.094			
607,148	1.39	41.148	0.426	41.574	5.471	4.537	10.008			
758,201	1.04	14.022	0.365	14.387	3.802	2.273	6.075			
869,587	0.96	27.513	0.510	28.023	5.362	3.414	8.776			
878,520	0.97	60.138	0.619	60.829	6.581	5.492	12.073			
918,840	0.88	59.094	0.679	59.773	6.104	4.483	10.592			
992,514	1.70									
1005,852	0.85	158.410	0.916	159.326	9.674	9.106	18.680			
1007,987	0.85				7.870	6.703	14.573			
1242,937	0.68				11.612	9.282	20.894			
2680,2500	0.35									

PROGRAM LISTINGS

A listing of the Munksgaard program suitably modified for the CDC 6000 series of computers will be found in Gignac.[3] Listings for CSKYDG2 and the Yale program will be found in Gignac[4] and Eisenstat et al.,[7] respectively. The reader is reminded that there are restrictions on the use and distribution of the Yale program.

OBSERVATIONS AND CONCLUSIONS

An examination of the times in the table indicates that the Yale program is the fastest by far of all three programs, ranging up to six times faster than the Munksgaard program (total time). The Munksgaard program itself can be four to five times faster than CSKYDG2 (total time). To some extent the time difference between the first two programs can be ascribed to the fact that the Yale program resequences the system prior to solution, while the Denmark program reorders while solving the system. In any event the first two programs, since they do everything in core, have a tremendous time advantage over CSKYDG2 which makes considerable use of random access storage.

However, a field length of 300000_8 CM was required to obtain some of the Yale and Munksgaard times in the table, even though the simplest of driving programs was used. Clearly the use of some device such as "overlay" would be required to use either the Yale or the Munksgaard program in practical work. These two programs are really suited for computers, such as the Texas Instruments Advanced Scientific Computer, which have abundant core storage, although the programs would have to be rewritten for the most part to obtain the full benefit of the computer's optimizing capability. For further testing on the CDC 6000 series these two programs should definitely be modified by the introduction of integer packing and unpacking subroutines to save core storage by cutting down the size of the integer arrays.

Since the CSKYDG2 program is profile oriented,[3] its performance is greatly improved when preceded by a preprocessor such as BANDIT[9,10] which resequences the system to minimize profile. For example consider the following data:

	N,M	SETUP	CSKYDG2	TOTAL
(from table)	209,185	4.789	23.140	27.929
(after BANDIT resequencing)	209,185	4.679	4.224	8.903
(from table)	221,188	6.413	20.138	26.551
(after BANDIT resequencing)	221,188	5.242	1.993	7.235

Since the Yale and Munksgaard programs are not profile oriented, their performance is not significantly affected by the BANDIT resequencing.

All three programs provided the same degree of accuracy for the computed solutions--some thirteen or so decimal places.

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